Chargino contributions to the CP asymmetry in $B\to\phi K_S$ decay

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We perform a model independent analysis of the chargino contributions to the CP asymmetry in the $B\to\phi K_S$ process. We use the mass insertion approximation method generalized by including the possibility of a light right stop. We find that the dominant effect is given by the contributions of the mass insertions $(\delta^u_{LL})_{32}$ and $(\delta^u_{RL})_{32}$ to the Wilson coefficient of the chromomagnetic operator. By considering both these contributions simultaneously, the CP asymmetry in the $B\to\phi K_S$ process is significantly reduced, and negative values, which are within the 1σ experimental range and satisfy the $b\to s\gamma$ constraints, can be obtained.

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The measurement of CP asymmetries in nonleptonic B decays plays a crucial role in testing the CP violation mechanism of the standard model (SM) and it is a powerful probe of new physics (NP) beyond the SM. The CP asymmetries are usually described by the time dependent rates $a_{f_{CP}}(t)$, for B^0 and $\overline{B}^0$ to a CP eigenstate f_{CP}:

$$a_{f_{CP}}(t)=\frac{\Gamma(\overline{B}^0(t)\to f_{CP})-\Gamma(B^0(t)\to f_{CP})}{\Gamma(\overline{B}^0(t)\to f_{CP})+\Gamma(B^0(t)\to f_{CP})},$$
$$=C_{f_{CP}}\cos\Delta M_{B_d}t+S_{f_{CP}}\sin\Delta M_{B_d}t \qquad (1)$$

where $C_{f_{CP}}$ and $S_{f_{CP}}$ represent the coefficients of direct and indirect CP violations respectively, and ΔM_{B_d} is the B^0 eigenstate mass difference.

The time dependent CP asymmetry $a_{J/\psi K_S}(t)$ in the B meson decay $B\to J/\psi K_S$ has been recently measured by the BaBar and Belle Collaborations, with an average of $S_{J/\psi K_S}=\sin 2\beta=0.734\pm0.034$ [1,2], showing the first evidence of CP violation in the B meson system in perfect agreement with the standard model predictions. This is expected, since the SM contribution is at the tree level.

For the decay $B\to\phi K_S$, where the same weak phase is measured, the situation is qualitatively different. The SM contribution is at the one-loop level, and one can expect crucial contributions from NP. The branching ratio for $B\to\phi K_S$ has recently been measured by both BaBar and Belle [3] with an average for the branching ratio of $\mathrm{BR}(B\to\Phi K_S)=(8.4^{+2.5}_{-2.1})\times10^{-6}$, which is slightly different from the SM prediction. However, the SM evaluation of $\mathrm{BR}(B\to\Phi K_S)$ is greatly affected by theoretical uncertainties in the evaluation of hadronic matrix elements, while they almost cancel out in the ratio of rates in the time dependent CP asymmetry.

Recently, the BaBar and Belle Collaborations [4,2] have also measured the time dependent CP asymmetry in the $B\to\phi K_S$ process, reporting an average value of $S_{\phi K_S}=-0.39\pm0.41$. In the SM, $S_{\phi K_S}$ is expected to give the same value of $\sin 2\beta$ as extracted from $S_{J/\psi K_S}$, up to terms of order $O(\lambda^2)$, where λ is the Cabibbo mixing. Thus, a comparison of the experimental results for $S_{J/\psi K_S}$ and $S_{\phi K_S}$ reveals a 2.7σ deviation from the SM prediction. If this discrepancy is confirmed with a better accuracy, it will be a clean signal of NP.

Due to the additional sources of flavor and CP violation beyond those of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, supersymmetric (SUSY) models are natural candidates for explaining the difference between the CP asymmetries $S_{\phi K_S}$ and $S_{J/\psi K_S}$. Recently, the gluino contributions to $S_{\phi K_S}$ have been analyzed in Refs. [5,6]. It was shown that gluino exchange can explain the experimental results for $S_{\phi K_S}$ without conflicting with the experimental constraints from $S_{J/\psi K_S}$ and the branching ratio $\mathrm{BR}(b\to s\gamma)$.

The main purpose of this article is to show that the chargino contributions to $S_{\phi K_S}$ can also be significant and can account for these recent measurements. We perform a model independent analysis by using the well known method of the mass insertion approximation [7], generalized by including the possibility of a light right top squark (right stop) in the otherwise almost degenerate squark spectrum. In our analysis, we take into account all the operators that contribute to the effective Hamiltonian for $\Delta B=1$ transitions, $H_{\mathrm{eff}}^{\Delta B=1}$, and provide analytical results for the corresponding leading Wilson coefficients.

Now we start our analysis of the SUSY contributions to the time dependent CP asymmetry in $B\to\phi K_S$ decay. In the following we will adopt the parametrization of the SM and SUSY amplitudes as in Ref. [5], namely,

$$\left(\frac{A^{\mathrm{SUSY}}}{A^{\mathrm{SM}}}\right)_{\phi K_S}\equiv R_\phi\ e^{i\theta_\phi}\ e^{i\delta_{12}}, \qquad (2)$$

where θ_ϕ is the SUSY CP violating phase, and δ_{12} is the strong (CP conserving) phase. In this case, the mixing CP asymmetry $S_{\phi K_S}$ takes the following form:

$$S_{\phi K_S}=\frac{\sin 2\beta+2R_\phi\cos\delta_{12}\sin(\theta_\phi+2\beta)+R_\phi^2\sin(2\theta_\phi+2\beta)}{1+2R_\phi\cos\delta_{12}\cos\theta_\phi+R_\phi^2}.$$

The most general amplitude for the $B\to\phi K_S$ process can be written as

$$\bar{A}(\phi K)=-\frac{G_F}{\sqrt{2}}\sum_{i=1}^{12}[C_i(\mu)+\tilde{C}_i(\mu)]\langle\phi\bar{K}^0|Q_i(\mu)|\bar{B}^0\rangle, \tag{3}$$

where Q_i are the operators that contribute to the effective Hamiltonian for $\Delta B=1$ transitions and $C_i(\mu)$ are the corresponding Wilson coefficients at the energy scale μ. The matrix elements $\langle\phi\bar{K}^0|Q_i|\bar{B}^0\rangle$ are calculated in the naive factorization approximation [8], and their expressions can be found in Ref. [5]. In this notation, $Q_{i=1-10}$ represent the four-fermion operators, and Q_{11} and Q_{12} the magnetic and chromomagnetic dipole operators, respectively. The Wilson coefficients $\tilde{C}_i$ are associated with the operators $\tilde{Q}_i$, which are obtained from Q_i by exchanging $\gamma_5\to-\gamma_5$ in their chiral structure; see Ref. [5] for their definition. In the SM, $\tilde{C}_i$ are chirally suppressed with respect to C_i by terms proportional to the light quark masses. However, in nonminimal SUSY extensions of the SM they can receive sizable contributions, for instance, from the gluino mediated penguin and box diagrams. On the other hand, the chargino contributions to $\tilde{C}_i$ are always suppressed by Yukawa couplings of the first two generations [9]. Thus, we can safely neglect $\tilde{C}_i$ contributions in our analysis.

The Wilson coefficients $C_i(\mu)$ at a lower scale $\mu\simeq O(m_b)$ can be extrapolated from the corresponding ones at high scale $C_i(\mu_W)$ as $C_i(\mu)=\Sigma_j\hat{U}_{ij}(\mu,\mu_W)C_j(m_W)$, where $\hat{U}_{ij}(\mu,\mu_W)$ is the QCD evolution matrix and $\mu_W\simeq m_W$. Since the operator Q_{12} is of order α_s, we include in our analysis the leading order (LO) corrections only for the effective Wilson coefficient $C_{12}(\mu)$, while for the remaining $C_{i=1-10}(\mu)$ we use the matrix $\hat{U}_{ij}(\mu,\mu_W)$ at next-to-leading order (NLO) in QCD and QED [10].

The chargino contributions to $C_i(\mu_W)$, corresponding to the effective Hamiltonian for $\Delta B=1$ transitions, have been calculated exactly (at one-loop) in Refs. [11] and [12]. Here we provide the results for these contributions, evaluated at the first order in the mass insertion approximation. By using the notation of Ref. [12] we obtain

$$F_\chi=\xi_{LL}R_F^{LL}+Y_t(\xi_{RL}R_F^{RL}+\xi_{LR}R_F^{LR})+Y_t^2\xi_{RR}R_F^{RR}, \tag{4}$$

where ξ_{AB} are given by $\xi_{LL}=\Sigma_{a,b}K_{a2}^\star K_{b3}(\delta_{LL}^u)_{ba}$, $\xi_{RR}=K_{32}^\star K_{33}(\delta_{RR}^u)_{33}$, $\xi_{RL}=\Sigma_a K_{a2}^\star K_{33}(\delta_{RL}^u)_{3a}$, and $\xi_{LR}=\Sigma_a K_{32}^\star K_{a3}(\delta_{LR}^u)_{a3}$. For the definition of the mass insertions $(\delta_{AB}^u)_{ij}$, see Ref. [7]. The same notation as in Ref. [12] has been used to relate the quantities F to the Wilson coefficients $C_{i=1-10}(\mu_W)$, while for the magnetic and chromomagnetic contributions we have $C_{11}(\mu_W)=M^\gamma$ and $C_{12}(\mu_W)=M^g$. Here, Y_t is the Yukawa coupling of the top quark and F refers to the photon penguins (D), Z penguins (C), gluon penguins (E), boxes with external down quarks ($B^{(d)}$) and up quarks ($B^{(u)}$), magnetic penguins (M^γ), and chromomagnetic (M^g) penguin diagrams. There are also contributions from box diagrams mediated by both gluino and chargino exchanges, which affect only $C_{i=1,2}(\mu_W)$, but their effect is negligible [12] and we will not include them in our analysis.

The detailed expressions for R_F, including contributions from chargino-gluino box diagrams, are given in the Appendix. Here we will just concentrate on the dominant contributions, which turn out to be due to the chromomagnetic (M^g) penguin and Z penguin (C) diagrams. In fact, for light SUSY particles ($\lesssim 1$ TeV), the contribution from the chromomagnetic penguin is one order and two orders of magnitudes larger than the corresponding ones from the Z penguin and other diagrams, respectively. However, in our numerical analysis we take into account all the contributions.

From Eq. (4), it is clear that the LR and RR contributions are suppressed by order λ^2 or λ^3. Since we will work in $O(\lambda)$ order, we can neglect them and simplify F_χ as

$$F_\chi=\xi_{LL}R_F^{LL}+Y_t\xi_{RL}R_F^{RL} \tag{5}$$

with $\xi_{LL}=(\delta_{LL}^u)_{32}+\lambda(\delta_{LL}^u)_{31}$ and $\xi_{RL}=(\delta_{RL}^u)_{32}+\lambda(\delta_{RL}^u)_{31}$. The functions R_F^{LL} and R_F^{RL} depend on the SUSY parameters through the chargino masses (m_{χ_i}), squark masses ($\tilde{m}$), and the entries of the chargino mass matrix. For instance, for magnetic (chromomagnetic) dipole penguins $R_{M^{\gamma(g)}}^{LL,RL}$, respectively, we have

$$R_{M^{\gamma,g}}^{LL}=\sum_i |V_{i1}|^2 x_{Wi}P_{M^{\gamma,g}}^{LL}(x_i)-Y_b\sum_i V_{i1}U_{i2}x_{Wi}\frac{m_{\chi_i}}{m_b}P_{M_{\gamma,g}}^{LR}(x_i),$$

$$R_{M^{\gamma,g}}^{RL}=-\sum_i V_{i1}V_{i2}^\star x_{Wi}P_{M_{\gamma,g}}^{LL}(x_i), \tag{6}$$

where Y_b is the Yukawa coupling of the bottom quark, $x_{Wi}=m_W^2/m_{\chi_i}^2$, $x_i=m_{\chi_i}^2/\tilde{m}^2$, and $\bar{x}_i=\tilde{m}^2/m_{\chi_i}^2$. The loop functions $P_{M_{\gamma,g}}^{LL(LR)}$ are given by

$$P_{M_\gamma}^{LL(LR)}(x)=-x\frac{d}{dx}\left(xF_{1(3)}(x)+\frac{2}{3}xF_{2(4)}(x)\right),$$

$$P_{M_g}^{LL(LR)}=-x\frac{d}{dx}[xF_{2(4)}(x)], \tag{7}$$

where the functions $F_i(x)$ can be found in Ref. [11]. Finally, U and V are the matrices that diagonalize the chargino mass

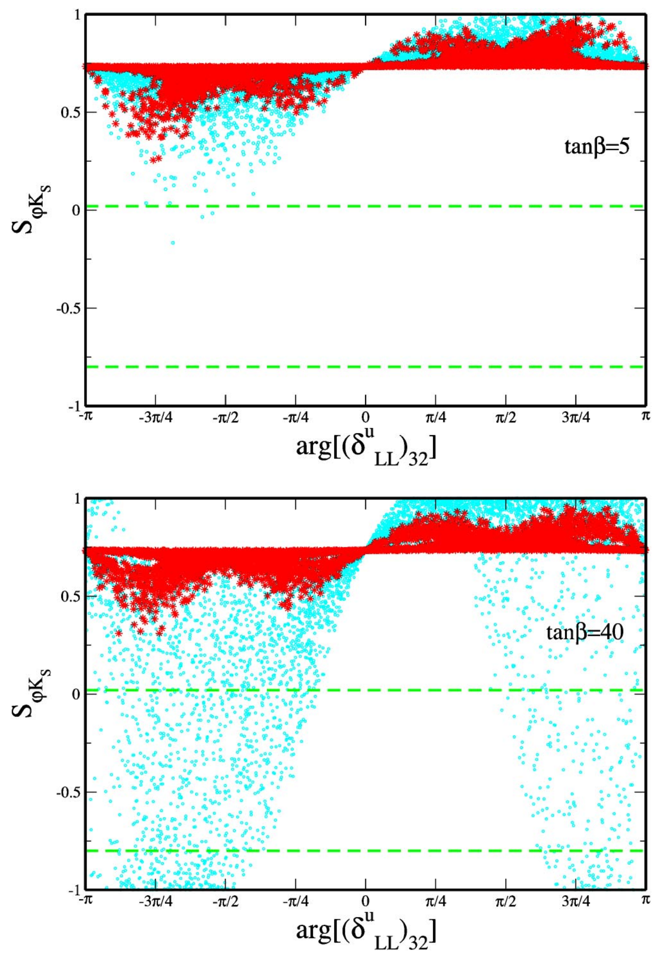

FIG. 1. $S_{\phi K_S}$ as a function of $\arg[(\delta^u_{LL})_{32}]$ for $\tan\beta=5{,}40$ and $\delta_{12}=0$ with the contribution of one mass insertion $|(\delta^u_{LL})_{32}|$. Darker points satisfy the constraints from BR($b\to s\gamma$), while lighter points do not.

matrix, defined as $U^* M_{\tilde{\chi}^+} V^{-1} = \mathrm{diag}(m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+})$, where we adopted the notation of Ref. [12] for the chargino mass matrix $M_{\tilde{\chi}^+}$.

Notice that the dependence on the Yukawa coupling of the bottom quark Y_b in Eq. (6) leads to enhancing C_{12} at large $\tan\beta$. Here, we also considered the case in which the mass of the right stop ($m_{\tilde{t}_R}$) is less than other squarks. In this case the functional form of Eq. (4) remains unchanged, while only the expressions for R_F^{RL} should be modified by replacing the functions inside $P^{LL,RL}_{M^{\gamma,g}}$ as

$$-x_i \frac{d}{dx_i} x_i F_a(x_i) \to \frac{1}{(x_t-1)}[x_{it} F_a(x_{it}) - x_i F_a(x_i)], \quad (8)$$

with index $a=1$–4, where $x_{it} = m^2_{\chi_i}/m^2_{\tilde{t}_R} = 1/\overline{x_{it}}$ and $x_t = m^2_{\tilde{t}_R}/\tilde{m}^2$.

We present our numerical results in Figs. 1–3, where the *CP* asymmetry $S_{\Phi K_S}$ is plotted versus the SUSY *CP* violating phase. In this analysis we work at fixed values of $\tan\beta$ and scan over all the relevant SUSY parameters: $\tilde{m}$, the weak gaugino mass M_2, the μ term, and $m_{\tilde{t}_R}$, and require that they satisfy the present experimental lower mass bounds, namely, the lightest chargino $m_\chi > 90$ GeV, heavy squarks $\tilde{m} > 300$ GeV, and light right stop $m_{\tilde{t}_R} > 150$ GeV. In addition, we scan over the real and imaginary parts of the corresponding mass insertions, by requiring that the $b\to s\gamma$ and B-$\bar{B}$ mixing constraints are satisfied. In our calculation we use the formula for the branching ratio (BR) $b\to s\gamma$ at the NLO in QCD, as provided in Ref. [13]. Indeed, the BR of $b\to s\gamma$ can easily be parametrized in terms of the SUSY contributions to the Wilson coefficients $C_{11}(\mu_W)$ and $C_{12}(\mu_W)$ given in Eq. (4) (see Ref. [13]).

In Figs. 1 and 2 we show the effects of one mass insertion per time, $(\delta^u_{LL})_{32}$ and $(\delta^u_{RL})_{32}$, evaluated at $\tan\beta=5$ and $\tan\beta=40$. θ_ϕ in Eq. (2) can be identified with the corresponding $\arg[(\delta^u_{AB})_{ij}]$. In these plots, the darker points are allowed by all experimental constraints, while the lighter points correspond to the points disallowed by BR($b\to s\gamma$) constraints at 95% C.L. In order to get the maximum effect for the negative values of *CP* asymmetry, we fixed the strong phase δ_{12} to be zero. We have not shown the contributions of the other mass insertions since they are subleading, being suppressed by terms of order λ.

As we can see from the results in Figs. 1 and 2, there is no chance with only one mass insertion to achieve large negative values for the *CP* asymmetry, especially in the case of $(\delta^u_{LL})_{32}$ (see Fig. 1). The main reason for $(\delta^u_{LL})_{32}$ is due to the $b\to s\gamma$ constraints[1] which are particularly sensitive to $\tan\beta$, while this is not the case for $(\delta^u_{RL})_{32}$. Moreover, as can be seen by comparing the scatter plots with $\tan\beta=5$ and $\tan\beta=40$ in Figs. 1 and 2, the allowed regions are not very sensitive to $\tan\beta$.

In Fig. 3 we show another example, where we take simultaneously both the mass insertions $(\delta^u_{LL})_{32}$ and $(\delta^u_{RL})_{32}$ per time, but assuming that their *CP* violating phase is the same. As can be seen from Fig. 3 there are points, allowed by $b\to s\gamma$ constraints, which can fit well inside the 1σ experimental region. In this case also the allowed regions are not very sensitive to $\tan\beta$.

In order to understand the behavior of these results, we look at the numerical parametrization of the ratios of amplitudes in terms of the relevant mass insertions. The main contribution to the SUSY amplitude is provided by the chromomagnetic dipole operator. For example, with $M_2=200$ GeV, $\mu=300$ GeV, $m_{\tilde{q}}=400$ GeV, $m_{\tilde{t}_R}=150$ GeV,

[1]In Ref. [14] chargino contributions to the *CP* asymmetry in $B\to\Phi K_S$ decay were analyzed in the limit of large $\tan\beta$ and in the exact case. We disagree with the results of that work regarding the effects of $b\to s\gamma$ constraints, which in our case strongly reduce the allowed chargino contributions in the region of large $\tan\beta$.

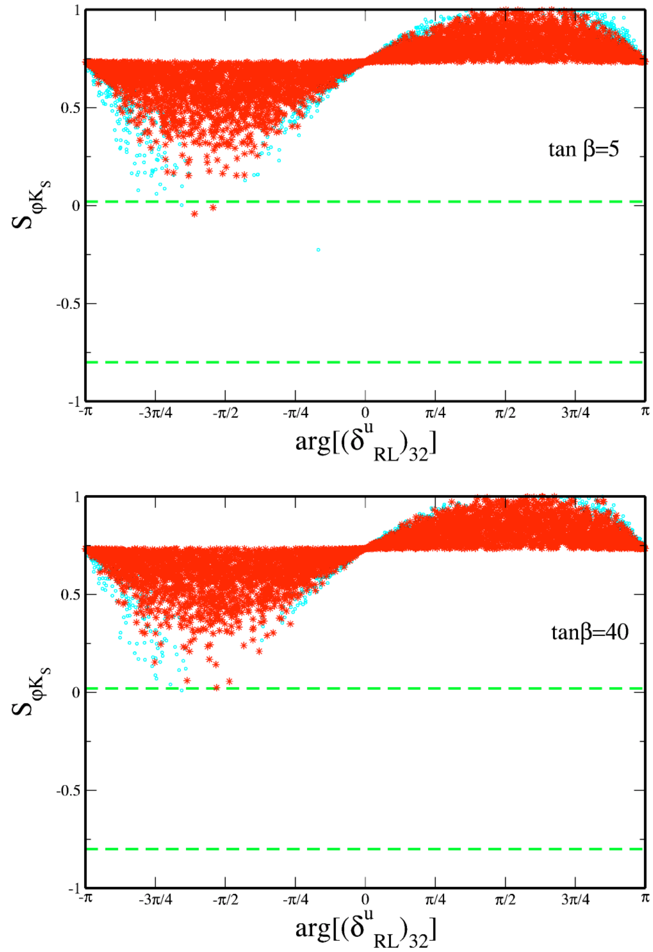

FIG. 2. As in Fig. 1, but for the mass insertion $(\delta^u_{RL})_{32}$.

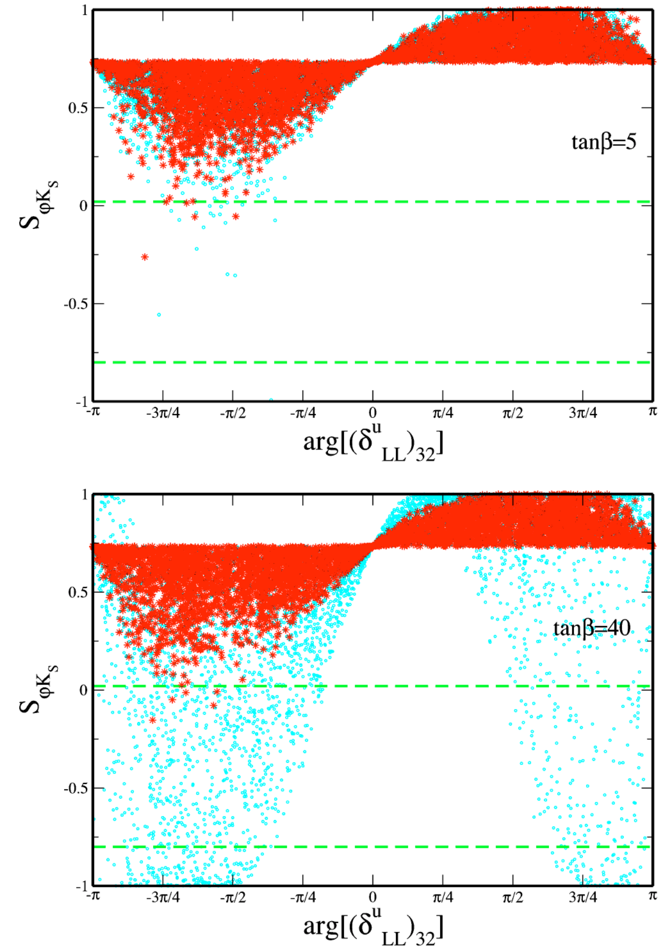

FIG. 3. As in Fig. 1, but for $\arg[(\delta^u_{LL})_{32}]=\arg[(\delta^u_{RL})_{32}]$ and with the contribution of two mass insertions $|(\delta^u_{RL})_{32}|$ and $|(\delta^u_{LL})_{32}|$.

and $\tan\beta=30$, we find $R^{RL}_C\simeq-0.033$, $R^{LL}_{Mg}\simeq-0.068$, while for all the others $R^{AB}_F\simeq O(10^{-3})$, and the amplitude ratio $R_A\equiv A^{\rm SUSY}/A^{\rm SM}$ is given by

$$R_A\simeq 0.37(\delta^u_{LL})_{31}+1.64(\delta^u_{LL})_{32}-0.05(\delta^u_{RL})_{31}-0.21(\delta^u_{RL})_{32}.$$

If we switch off the chromomagnetic dipole operator, the coefficients of the mass insertions δ^u_{LL} are significantly reduced, while the coefficients of δ^u_{RL} are slightly changed, and R_A takes the form

$$R_A\simeq-0.003(\delta^u_{LL})_{31}-0.014(\delta^u_{LL})_{32} -0.045(\delta^u_{RL})_{31}-0.2(\delta^u_{RL})_{32}.$$

The chromomagnetic contributions from R^{LL}_{Mg} are enhanced by $\tan\beta$, due to the term proportional to Y_b. For instance, for $\tan\beta\sim10$, the value of R^{LL}_{Mg} is reduced to $R^{LL}_{Mg}\simeq-0.023$, while R^{RL}_C is slightly increased to $R^{RL}_C\simeq-0.033$, and the amplitude ratio becomes

$$R_A\simeq 0.12(\delta^u_{LL})_{31}+0.54(\delta^u_{LL})_{32}-0.05(\delta^u_{RL})_{31}-0.21(\delta^u_{RL})_{32}.$$

Furthermore, with heavy SUSY particles ($m_{\tilde q}\sim1$ TeV), the Z penguin diagram would provide the dominant contributions to F_χ, since R^{RL}_C tends to a constant value of order -0.05. This effect clearly shows the phenomenon of nondecoupling of the chargino contribution to the Z penguin [15].

We stress that the contribution of $(\delta^u_{LL})_{32}$ to the chromomagnetic dipole operator, which leads to the dominant contribution to $S_{\phi K_S}$, is strongly constrained by $b\to s\gamma$ [which is particularly sensitive to $C_{11}(\mu_W)$]. This is due to the fact that $(\delta^u_{LL})_{32}$ gives almost the same contribution to both $C_{11}(\mu_W)$ and $C_{12}(\mu_W)$, as can be seen from Eq. (6). This is not the case for gluino exchanges, since there the contributions to the chromomagnetic dipole operator are enhanced by color factors with respect to the magnetic dipole ones, allowing large contributions to C_{12} while respecting the $b\to s\gamma$ constraints [16]. Regarding the effects of $(\delta^u_{RL})_{31}$ and $(\delta^u_{LL})_{31}$, their contributions to $S_{\phi K_S}$ are quite small since they are mostly constrained by ΔM_B and $\sin2\beta$ [9].

For the above set of input parameters, the $b\to s\gamma$ limits impose $|(\delta^u_{LL})_{32}|<0.58$. Thus, the maximum individual mass insertion contributions are given by $|A^{\rm SUSY}_{LL32}/A^{\rm SM}|<0.31$ and $|A^{\rm SUSY}_{RL32}/A^{\rm SM}|<0.21$. This shows that after imposing the $b\to s\gamma$ constraints the contribution from $(\delta^u_{LL})_{32}$ is of the same order as the contribution from $(\delta^u_{RL})_{32}$. However, each of them leads to $R_\phi\sim0.4$ at most, so even if $\sin\theta_\phi\sim-1$, one can reduce $S_{\phi K_S}$ from the SM prediction $\sin 2\beta$ to 0.2 and it is not possible with one mass insertion contribution to reach negative *CP* asymmetry. Nevertheless, by considering the contributions from both $(\delta^u_{LL})_{32}$ and $(\delta^u_{RL})_{32}$ simultaneously, R_ϕ can become larger and values of the order of $S_{\phi K_S} \simeq -0.2$ can be achieved.

It is worth mentioning that we have also considered the BR of $B^0 \to \phi K^0$ decay and ensured that the SUSY effects do not violate the experimental limits observed by BaBar and Belle [3].

Let us emphasize that generally in supersymmetric models the lighter chargino is expected to be one of the lightest sparticles. Thus, it can be expected to contribute significantly in the one-loop processes. Although the gluino contribution to the studied asymmetry can be very large, on the other hand the gluino in many models is one of the heaviest SUSY partners and thus its contribution may be considerably reduced.

Finally, we would like to stress that our results are consistent with the QCD factorization (QCDF) method [17] within a theoretical error. However, the computation in QCDF is not complete since at present the calculation of annihilation diagrams is missing, since these are higher order in α_s. We have found that the chromomagnetic contributions, from annihilation diagrams, play a crucial role in SUSY analysis, while they are small in the SM. Thus, there is no consistent way at the moment to implement a QCDF calculation for SUSY analysis. However, after this work was completed, new significant theoretical results in QCDF appeared in [18], where attempts at completing the calculation of the relevant hard scattering in QCDF are provided. The implication of this new calculation for our process will be considered in a forthcoming paper.

To conclude, we have studied the chargino contributions to the *CP* asymmetry $S_{\phi K_S}$ and showed that, although the experimental limits on $b \to s\gamma$ impose stringent constraints on the parameter space, it is still possible to reduce $S_{\phi K_S}$ significantly, and negative values within the 1σ experimental range can be obtained.

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APPENDIX

Here we provide the analytical results for the expressions R_F and $\overline{R}_F$ appearing in Eq. (4), which are given by

$$R_D^{LL} = \sum_{i=1,2} |V_{i1}|^2 x_{Wi} P_D(x_i),$$

$$R_D^{RL} = -\sum_{i=1,2} V_{i2}^\star V_{i1} x_{Wi} P_D(x_i),$$

$$R_D^{RR} = \sum_{i=1,2} |V_{i2}|^2 x_{Wi} P_D(x_i),$$

$$R_D^{LR} = (R_D^{RL})^\star,$$

$$R_E^{LL} = \sum_{i=1,2} |V_{i1}|^2 x_{Wi} P_E(x_i),$$

$$R_E^{RL} = -\sum_{i=1,2} V_{i2}^\star V_{i1} x_{Wi} P_E(x_i),$$

$$R_E^{RR} = \sum_{i=1,2} |V_{i2}|^2 x_{Wi} P_E(x_i),$$

$$R_E^{LR} = (R_E^{RL})^\star,$$

$$R_C^{LL} = \sum_{i=1,2} |V_{i1}|^2 P_C^{(0)}(\overline{x_i}) + \sum_{i,j=1,2} \Big[U_{i1} V_{i1} U_{j1}^\star V_{j1}^\star P_C^{(2)}(x_i, x_j) + |V_{i1}|^2 |V_{j1}|^2 \Big(\frac{1}{8} - P_C^{(1)}(x_i, x_j) \Big) \Big],$$

$$R_C^{RL} = -\frac{1}{2} \sum_{i=1,2} V_{i2}^\star V_{i1} P_C^{(0)}(\overline{x_i}) - \sum_{i,j=1,2} V_{j2}^\star V_{i1} [U_{i1} U_{j1}^\star P_C^{(2)}(x_i, x_j) + V_{i1}^\star V_{j1} P_C^{(1)}(x_i, x_j)],$$

$$R_C^{LR} = (R_C^{RL})^\star,$$

$$R_C^{RR} = \sum_{i,j=1,2} V_{j2}^\star V_{i2} [U_{i1} U_{j1}^\star P_C^{(2)}(x_i, x_j) + V_{i1}^\star V_{j1} P_C^{(1)}(x_i, x_j)],$$

$$R_{B^u}^{LL} = 2 \sum_{i,j=1,2} V_{i1} V_{j1}^\star U_{i1} U_{j1}^\star x_{Wj} \times \sqrt{x_{ij}} P_B^u(\overline{x_j}, x_{ij}),$$

$$R_{B^u}^{RL} = -2 \sum_{i,j=1,2} V_{i1} V_{j2}^\star U_{i1} U_{j1}^\star x_{Wj} \times \sqrt{x_{ij}} P_B^u(\overline{x_j}, x_{ij}),$$

$$R_{B^u}^{LR} = (R_{B^u}^{RL})^\star$$

$$R_{B^u}^{RR} = 2 \sum_{i,j=1,2} V_{i2} V_{j2}^\star U_{i1} U_{j1}^\star x_{Wj} \times \sqrt{x_{ij}} P_B^u(\overline{x_j}, x_{ij}),$$

$$R_{B^d}^{LL} = \sum_{i,j=1,2} |V_{i1}|^2 |V_{j1}|^2 x_{Wj} P_B^d(\overline{x_j}, x_{ij}),$$

$$R^{RL}_{B^d} = -\sum_{i,j=1,2} V^\star_{i2} V_{i1} |V_{j1}|^2 x_{Wj} P^d_B(\overline{x_j}, x_{ij}),$$

$$R^{LR}_{B^d} = (R^{RL}_{B^d})^\star$$

$$R^{RR}_{B^d} = \sum_{i,j=1,2} V^\star_{i2} V_{i1} V^\star_{j1} V_{j2} x_{Wj} P^d_B(\overline{x_j}, x_{ij}),$$

$$R^{LL}_{M^{\gamma,g}} = \sum_i |V_{i1}|^2 x_{Wi} P^{LL}_{M^{\gamma,g}}(x_i) - Y_b \sum_i V_{i1} U_{i2} x_{Wi} \frac{m_{\chi_i}}{m_b} P^{LR}_{M_{\gamma,g}}(x_i),$$

$$R^{LR}_{M^{\gamma,g}} = -\sum_i V^\star_{i1} V_{i2} x_{Wi} P^{LL}_{M_{\gamma,g}}(x_i) + Y_b \sum_i V_{i2} U_{i2} x_{Wi} \frac{m_{\chi_i}}{m_b} P^{LR}_{M_{\gamma,g}}(x_i),$$

$$R^{RL}_{M^{\gamma,g}} = -\sum_i V_{i1} V^\star_{i2} x_{Wi} P^{LL}_{M_{\gamma,g}}(x_i),$$

$$R^{RR}_{M^{\gamma,g}} = \sum_i |V_{i2}|^2 x_{Wi} P^{LL}_{M_{\gamma,g}}(x_i), \tag{A1}$$

where $x_{Wi} = m_W^2/m_{\chi_i}^2$, $x_i = m_{\chi_i}^2/\tilde{m}^2$, $\overline{x_i} = \tilde{m}^2/m_{\chi_i}^2$, and $x_{ij} = m_{\chi_i}^2/m_{\chi_j}^2$. The expressions for the functions $P_{E,D,C}$, $P_B^{(u,d)}$, $P^{LL}_{M_{\gamma,g}}$, and $P^{LR}_{M_{(\gamma,g)}}$, are given in the next subsection.

There are other contributions coming from box diagrams, where both charginos and gluinos are exchanged ($B^{u,c}_{\tilde{g}}$), which cannot be expressed in the same form as Eq. (4). We provide below the results for these contributions, which affect only the Wilson coefficients $C^{(u,c)}_{1,2}(\mu_W)$ as

$$C_1^{(u,c)}(\mu_W) = \frac{\alpha_s(m_W)}{16\pi}(14 - B^{(u,c)}_{\tilde{g}}),$$

$$C_2^{(u,c)}(\mu_W) = 1 + \frac{\alpha_s(m_W)}{48\pi} B^{(u,c)}_{\tilde{g}}, \tag{A2}$$

where

$$B^u_{\tilde{g}} = \left[\sum_a K^\star_{a2} K_{13} (\delta^u_{LL})_{1a}\right] R^{LL}_{\tilde{g}}(u) + \left[\sum_a K^\star_{12} K_{a3} (\delta^u_{LL})_{a1}\right] [R^{LL}_{\tilde{g}}(u)]^\star + \left[\sum_a K^\star_{1a} K_{13} (\delta^d_{LL})_{a2}\right] R^{LL}_{\tilde{g}}(d) + \left[\sum_a K^\star_{12} K_{1a} (\delta^d_{LL})_{3a}\right] [R^{LL}_{\tilde{g}}(d)]^\star + \left[\sum_a K^\star_{12} K_{33} (\delta^u_{RL})_{31}\right] Y_t R^{RL}_{\tilde{g}}, \tag{A3}$$

$$B^c_{\tilde{g}} = \left[\sum_a K^\star_{a2} K_{23} (\delta^u_{LL})_{2a}\right] R^{LL}_{\tilde{g}}(u) + \left[\sum_a K^\star_{22} K_{a3} (\delta^u_{LL})_{a2}\right] [R^{LL}_{\tilde{g}}(u)]^\star + \left[\sum_a K^\star_{2a} K_{23} (\delta^d_{LL})_{a2}\right] R^{LL}_{\tilde{g}}(d) + \left[\sum_a K^\star_{22} K_{2a} (\delta^d_{LL})_{3a}\right] [R^{LL}_{\tilde{g}}(d)]^\star + \left[\sum_a K^\star_{22} K_{33} (\delta^u_{RL})_{32}\right] Y_t R^{RL}_{\tilde{g}}, \tag{A4}$$

and the functions R_i are given by

$$R^{LL}_{\tilde{g}}(u) = 4 x_{W\tilde{g}} \sum_{i=1,2} \left[|V_{i1}|^2 P^d_B(z_i, y) + 2 U_{i1} V_{i1} \left(\frac{m_{\chi_i}}{m_{\tilde{g}}}\right) P^u_B(z_i, y)\right], \tag{A5}$$

$$R^{LL}_{\tilde{g}}(d) = 4 x_{W\tilde{g}} \sum_{i=1,2} \left[|U_{i1}|^2 P^d_B(z_i, y) + 2 U^\star_{i1} V^\star_{i1} \left(\frac{m_{\chi_i}}{m_{\tilde{g}}}\right) P^u_B(z_i, y)\right], \tag{A6}$$

$$R^{RL}_{\tilde{g}} = -4 x_{W\tilde{g}} \sum_{i=1,2} \left[V_{i1} V^\star_{i2} P^d_B(z_i, y) + 2 V^\star_{i2} U^\star_{i1} \left(\frac{m_{\chi_i}}{m_{\tilde{g}}}\right) P^u_B(z_i, y)\right] \tag{A7}$$

with $x_{W\tilde{g}} = m_W^2/m_{\tilde{g}}^2$, $z_i = m_{\chi_i}^2/m_{\tilde{g}}^2$, and $y = \tilde{m}^2/m_{\tilde{g}}^2$. In obtaining the above results in Eqs. (A3), (A4) we neglect terms of the order of the $O(Y_b)$.

Loop functions

Here we provide the expressions for the loop functions of penguin $P_{D,E,C}$, box $P_B^{(u,d)}$, and magnetic and chromomagnetic penguin diagrams $P^{LL}_{M_{\gamma,g}}$, and $P^{LR}_{M_{\gamma,g}}$, respectively, which enter in Eqs. (A1), and (A5)–(A7):

$$P_D(x)=\frac{2x[-22+60x-45x^2+4x^3+3x^4-3(3-9x^2+4x^3)\log x]}{27(1-x)^5},$$

$$P_E(x)=\frac{x(-1+6x-18x^2+10x^3+3x^4-12x^3\log x)}{9(1-x)^5},$$

$$P_C^{(0)}(x)=\frac{x(3-4x+x^2+2\log x)}{8(1-x)^3},$$

$$P_C^{(1)}(x,y)=\frac{1}{8(x-y)}\left[\frac{x^2(x-1-\log x)}{(x-1)^2}-\frac{y^2(y-1-\log y)}{(y-1)^2}\right],$$

$$P_C^{(2)}(x,y)=\frac{\sqrt{xy}}{4(x-y)}\left[\frac{x(x-1-\log x)}{(x-1)^2}-\frac{y(y-1-\log y)}{(y-1)^2}\right],$$

$$P_B^u(x,y)=\frac{-y-x(1-3x+y)}{4(x-1)^2(x-y)^2}-\frac{x(x^3+y-3xy+y^2)\log x}{2(x-1)^3(x-y)^3}+\frac{xy\log y}{2(x-y)^3(y-1)},$$

$$P_B^d(x,y)=-\frac{x[3y-x(1+x+y)]}{4(x-1)^2(x-y)^2}-\frac{x[x^3+(x-3)x^2y+y^2]\log x}{2(x-1)^3(x-y)^3}+\frac{xy^2\log y}{2(x-y)^3(y-1)},$$

$$P^{LL}_{M_\gamma}(x)=-x\frac{d}{dx}\left(xF_1(x)+\frac{2}{3}xF_2(x)\right),$$

$$P^{LR}_{M_\gamma}(x)=-x\frac{d}{dx}\left(xF_3(x)+\frac{2}{3}xF_4(x)\right),$$

$$P^{LL}_{M_g}(x)=-x\frac{d}{dx}[xF_2(x)],$$

$$P^{LR}_{M_g}(x)=-x\frac{d}{dx}[xF_4(x)], \tag{A8}$$

where the functions $F_i(x)$ are provided in Ref. [11].

Light right stop

Here we generalize the above formulas for the case in which the right stop is lighter than other squarks. Notice that this will modify only the expressions for R_F^{RL} and R_F^{RR}, since the light right stop does not affect R_F^{LL}. In the case of R_F^{RR}, the functional forms of R_F^{RR} remain unchanged, while the arguments of the functions involved are changed as $x_i \to x_{it}$ and $\overline{x_i} \to \overline{x_{it}}$. In the case of R_F^{LR} and R_F^{RL}, the analytical expressions for the loop functions of penguin $P_{D,E,C}$, box $P_B^{(u,d)}$, and magnetic and chromomagnetic penguin diagrams $P^{LL}_{M_{\gamma,g}}$ and $P^{LR}_{M_{\gamma,g}}$, respectively, should be changed as follows:

$$P_D(x_i,x_{it})=\frac{2}{(x_t-1)}[x_{it}D_\chi(x_{it})-x_iD_\chi(x_i)],$$

$$P_E(x_i,x_{it})=\frac{2}{(x_t-1)}[x_{it}E_\chi(x_{it})-x_iE_\chi(x_i)],$$

$$P_C^{(1,2)}(x_i,x_{it},x_j,x_{jt})=\frac{2}{(x_t-1)}[C_\chi^{(1,2)}(x_{jt},x_{it})-C_\chi^{(1,2)}(x_j,x_i)],$$

$$P_C^{(0)}(\overline{x_i},\overline{x_{it}}) = \frac{4}{(x_t-1)}[C_\chi^{(1)}(\overline{x_{it}},\overline{x_i}) - C_\chi^{(1)}(\overline{x_i},\overline{x_i})],$$

$$P_B^{(u)}(\overline{x_j},\overline{x_{jt}},x_{ij}) = \frac{1}{2(x_t-1)}[B_\chi^{(u)}(\overline{x_{jt}},\overline{x_j},x_{ij}) - B_\chi^{(u)}(\overline{x_j},\overline{x_j},x_{ij})],$$

$$P_B^{(d)}(\overline{x_j},\overline{x_{jt}},x_{ij}) = -\frac{1}{2(x_t-1)}[B_\chi^{(d)}(\overline{x_{jt}},\overline{x_j},x_{ij}) - B_\chi^{(d)}(\overline{x_j},\overline{x_j},x_{ij})],$$

$$P_{M_\gamma}^{LL}(x_i,x_{it}) = \frac{1}{(x_t-1)}\left[x_{it}\left(F_1(x_{it}) + \frac{2}{3}F_2(x_{it})\right) - x_i\left(F_1(x_i) + \frac{2}{3}F_2(x_i)\right)\right],$$

$$P_{M_\gamma}^{LR}(x_i,x_{it}) = \frac{1}{(x_t-1)}\left[x_{it}\left(F_3(x_{it}) + \frac{2}{3}F_4(x_{it})\right) - x_i\left(F_3(x_i) + \frac{2}{3}F_4(x_i)\right)\right],$$

$$P_{M_g}^{LL}(x_i,x_{it}) = \frac{1}{(x_t-1)}[x_{it}F_2(x_{it}) - x_iF_2(x_i)],$$

$$P_{M_g}^{LR}(x_i,x_{it}) = \frac{1}{(x_t-1)}[x_{it}F_4(x_{it}) - x_iF_4(x_i)], \tag{A9}$$

where $x_i = m_{\chi_i}^2/\tilde{m}^2$, $\overline{x_i} = \tilde{m}^2/m_{\chi_i}^2$, $x_{it} = m_{\chi_i}^2/m_{\tilde{t}_R}^2$, $\overline{x_{it}} = m_{\tilde{t}_R}^2/m_{\chi_i}^2$, $x_{ij} = m_{\chi_i}^2/m_{\chi_j}^2$, and $x_t = m_{\tilde{t}_R}^2/\tilde{m}^2$. The functions $D_\chi, C_\chi, E_\chi, C_\chi^{(1,2)}, B_\chi^{(u,d)}$, and F_i are provided in Ref. [12] and Ref. [11].

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